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JAMES A. BRIDGE, JR.

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by

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Submitted in partial fulfillment of
the requirement for the degree of

MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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MASTER OF SCIENCE
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MECHANICAL ENGINEERING
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United States Naval Postgraduate School

ABSTRACT

Tests conducted last year at the United States Naval Postgraduate School, Monterey, California, resulted in an evaluation of the effect of two stress-raising discontinuities acting at a common point for the case of torsion. This paper represents an effort to further the investigation of the superposition of two stress raisers; in particular, for the case of reversed bending.

The discontinuities consisted of a radial hole and a course machined surface. The results, using AISI 4340 steel of a nominal tensile strength of 200 ksi are presented. The data were correlated and compared with those of two other investigators, and is presented in tabular as well as graphical form.

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APPENDICES

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LIST OF ILLUSTRATIONS

1. INTRODUCTION

Many years have passed since the effect of stress concentrations upon the endurance strength of materials has been recognized and incorporated into the design of machine parts. So much data have been accumulated and evaluated that it is possible to find a chart or at least a particular reference which will assist in predicting reasonably well the reduction in endurance strength caused by a particular discontinuity. However, if a machine member in service is to be subjected to the simultaneous action of two stress raisers, then the designer is faced with a decision without experimental evidence, as the literature is almost barren of information pertaining to this cumulative effect.

MacGregor [1]* in 1936 cited an instance in which a fatigue failure of a shaft was initiated at a rough place on the surface of a drilled hole. The machinist had polished the shaft but forgot the hole, so the stress raising effect was added to that of the hole itself.

In 1938, Dolan [2] investigated the effects of fresh water corrosion upon the endurance strength of steel specimens containing holes or fillets, where the percentage reduction in endurance strength caused by the action of the water represents an equivalent corrosion stress concentration factor.

A combination of fillet and groove was investigated by Mowbray [3] in 1953. A search of the literature revealed no

* Numbers enclosed in brackets refer to references on page 34.

[illegible]

further information along these lines until the investigation conducted by Guhse [4] in 1960.

It was the intent of this investigation to determine the effect in reversed bending of superposing the discontinuity caused by a coarse machined surface upon that caused by a small radial hole.

The tests described herein were conducted at the United States Naval Postgraduate School, Monterey, California, during the period of December 1960 - May 1961 under the supervision of Professor Virgil M. Faires, with the aim of shedding more light upon the effect of superposed discontinuities.

2. DESCRIPTION OF TESTING MACHINE

The machine used for the tests was a Krouse high-speed repeated-stress machine (Serial #580). It is of the mechanical, non-resonant constant load, single-end cantilever design, with a scale beam loading system for producing the desired stress level. (See Figure 1) The speed of the machine, controlled by positioning a variable transformer, V, Figure 1, is continuously variable from 0-12,000 RPM. Since the manufacturer recommends speeds not in excess of 10,000 RPM, all of the tests were conducted at a speed of 9500 RPM ($\pm$ 500 RPM). Minor variations in speed were caused by the normal voltage fluctuations.

In Figure 1 is shown the testing machine with one of the specimens gripped between the collets, C. The sliding weight, W, can be positioned along the scale, S, which is graduated in inch-pounds of moment produced at the minimum section of the specimen. The weight-beam reading is read at the index, I, on the sliding weight. The scale is graduated from 0-140 inch-pounds, with 1 inch-pound per graduation. The sliding weight is locked in place at the desired position along the scale by means of the locking screw, L. The machine is equipped with a micro-switch, M, to stop it when a specimen fails. The counter, N, set at zero at the beginning of each run, indicated the number of cycles endured by each specimen.

An effort was made to start and increase the speed of the testing machine in a uniform manner, such that the operating speed was reached at approximately the same time for each specimen.

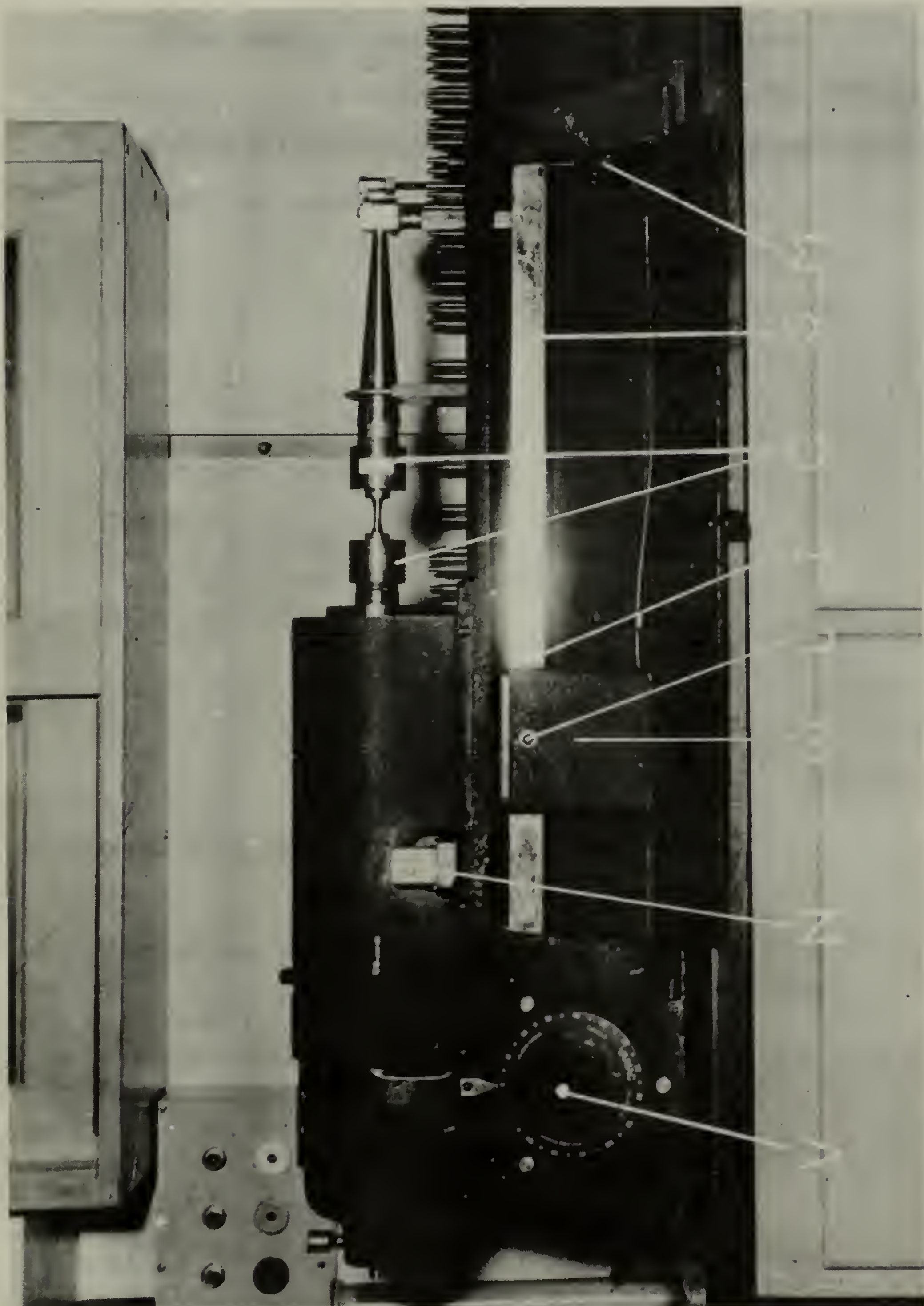


Figure 1

Because there was no practical accurate means of measuring amplitude of displacement of the specimens in the testing machine, the micro-switch was adjusted so that actual fracture of a specimen stopped the machine.

3. TYPE OF MATERIAL AND SPECIMEN PREPARATION

The material chosen for these tests was AISI 4340 steel, a widely used triple alloy steel with high hardenability. It is relatively free from temper brittleness and can be machined at relatively high hardness levels. The certified chemical composition is: carbon, 0.395%; manganese, 0.73%; phosphorus, 0.010%; sulfur, 0.017%; silicon, 0.31%; chromium, 0.75%; nickel, 1.65%; molybdenum, 0.23%. The grain size is 7. Certified to be from the same heat, the material was delivered in the annealed condition and consisted of eight rods of 7/16-in. diameter, 3 ft. in length.

It was decided to use a slightly modified type of specimen as recommended [5] for use with the Krouse machine (see Figure 2). The modification consisted of reducing the radius of curvature of the test section from 2 in. to 1.5 in. The recommended minimum radius is six times the minimum diameter [6], in this case, $6 \times 0.2 = 1.2$ in. The 1.5 in. radius of curvature meets this requirement.

The specimens were cut 3±1/64 in. in length and numbered 1 through 105. No effort was made to distinguish between rods or as to the location each specimen occupied in a rod, mainly because the standard 20-foot length had been cut without identifying markings in order to facilitate shipment. Also, in a similar investigation performed by Guhse [4], there was no indication that the positions the specimens had occupied in the rods had any significance in the interpretation of his results.

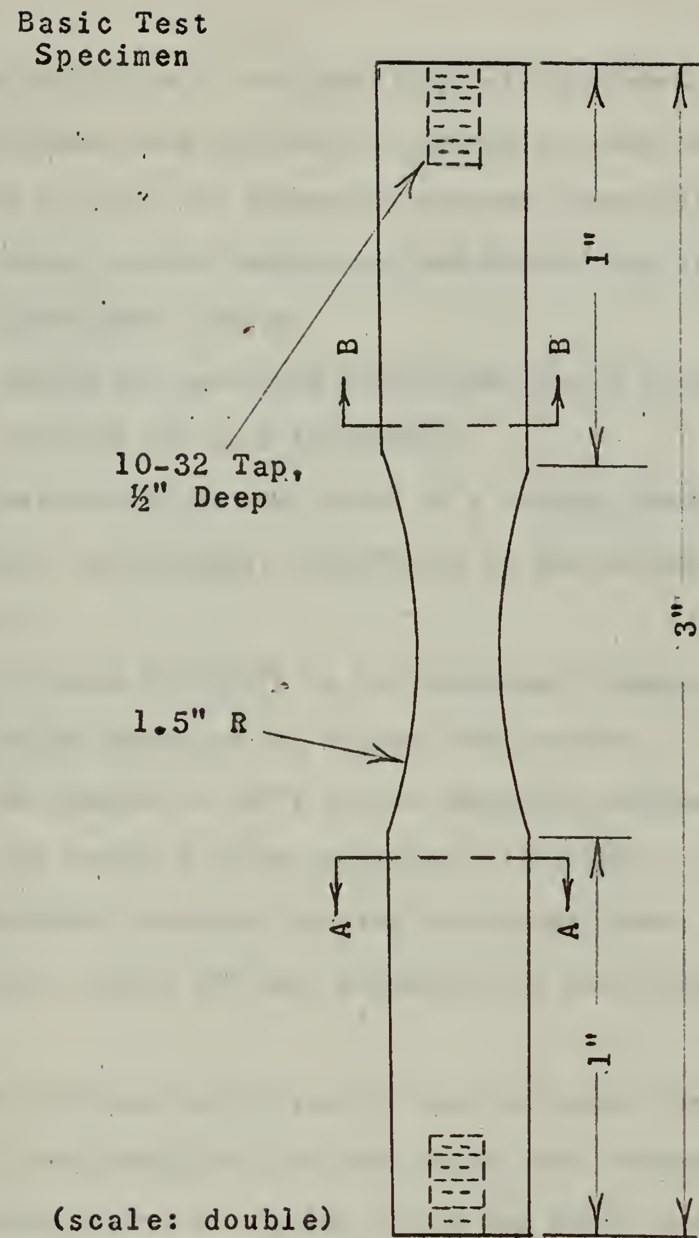

Basic Test
Specimen
10-32 Tap,
½" Deep
1.5" R
(scale: double)
B
B
A
A
1"
1"
3"

Figure 2

The facilities of the Metallurgical Department of the Post-Graduate School were utilized to perform the heat treatment. Care was taken to avoid the effects of surface decarburization by:

1. using cracked ammonia gas and natural gas in a controlled atmosphere furnace.
2. having the specimens 0.030-0.040 in. in diameter over-size for the heat treatment.

It was decided to heat treat to a nominal tensile strength of 200 ksi; accordingly, from Figure 3, the following procedure was used:

1. $1\frac{1}{2}$ hours at 1530°F in the hardening furnace.
2. rapid quench in oil to room temperature.
3. 30 minutes at 860°F in the tempering furnace.
4. air cooled to room temperature in still air.

Throughout the heat treating operations, every effort was made to use exactly the same procedure for each group of specimens.

The specimens were placed in the hardening furnace ten at a time in sub-groups of five each which were fastened at the ends with machine screws to a piece of 3/4 in. angle iron with 1 inch between specimen centers. The purpose of this fixture was to facilitate handling of the specimens.

In order to determine if the maximum as-quenched hardness had been obtained, tests were made on the surface and approximately 0.010 in. below the surface. Failure to achieve this condition before tempering can result in considerable loss of tensile strength [7].

AISI-4340

(Oil Quenched)

PROPERTIES CHART

(Single Heat Results)

.530" Rd. SIZE TREATED .505" Rd. SIZE TESTED	Ac_1 1350°F. Ac_3 1415°F.	Ar_3 890°F. Ar_1 720°F.	C. .38/.43	Mn. .60/.80	P. Max. .04	S. Max. .04	Si. .20/.35	Ni. 1.65/2.00	Cr. .70/.90	Mo. .20/.30	Grain Size
HEAT TESTED			.41	.67	.023	.018	.26	1.77	.78	.26	6-8

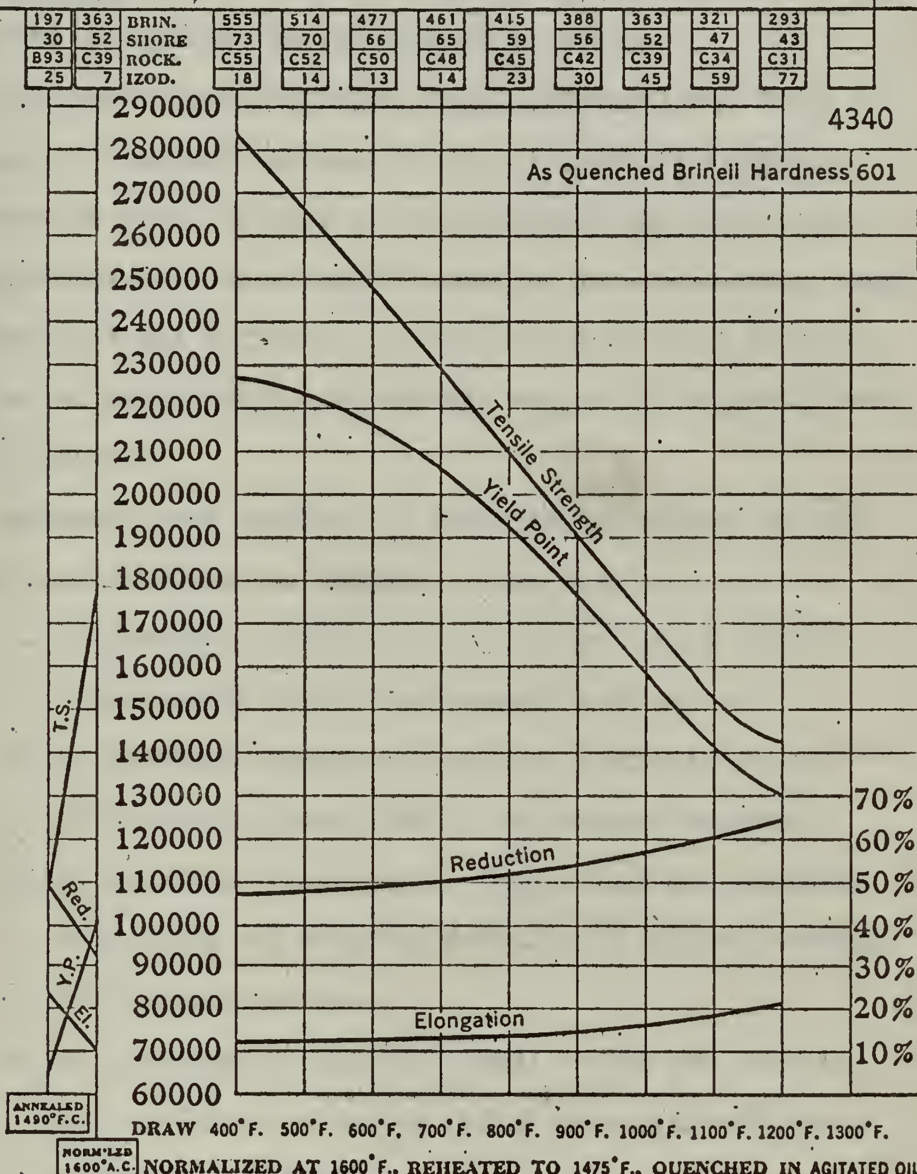


Figure 3

An average quenched hardness of 55-57 Rockwell C was obtained, which closely agrees with values in Figure 3. At the same time it was noted that the difference in hardness between the surface and 0.010 in. below the surface was no more than one point, and it seemed safe to conclude that the controlled atmosphere furnace was effective in avoiding surface decarburization.

The average tempered hardness of all specimens was found to be 39.7 R_c, or a nominal hardness of 40. A hardness frequency distribution bar chart (Figure 4a) approximates the statistical "bell-shaped" curve. The standard deviation σ, was computed, and it was found that all but three specimens fell within 2σ limits, which is an indication that unassignable causes of variation were largely eliminated.

The specimens were machined to final specifications in the four basic configurations as follows: (see Figure 5)

A. 25 specimens polished to a surface finish of 10 micro-inches RMS at the reduced section.

B. 25 specimens coarse machined to a surface finish of 500 micro-inches RMS at the reduced section.

C. 25 specimens identical to type A with the addition of a 0.025 in. diameter radial hole drilled through the minimum section.

D. 30 specimens identical to type B with the addition of a 0.025 in. diameter radial hole drilled through the minimum section.

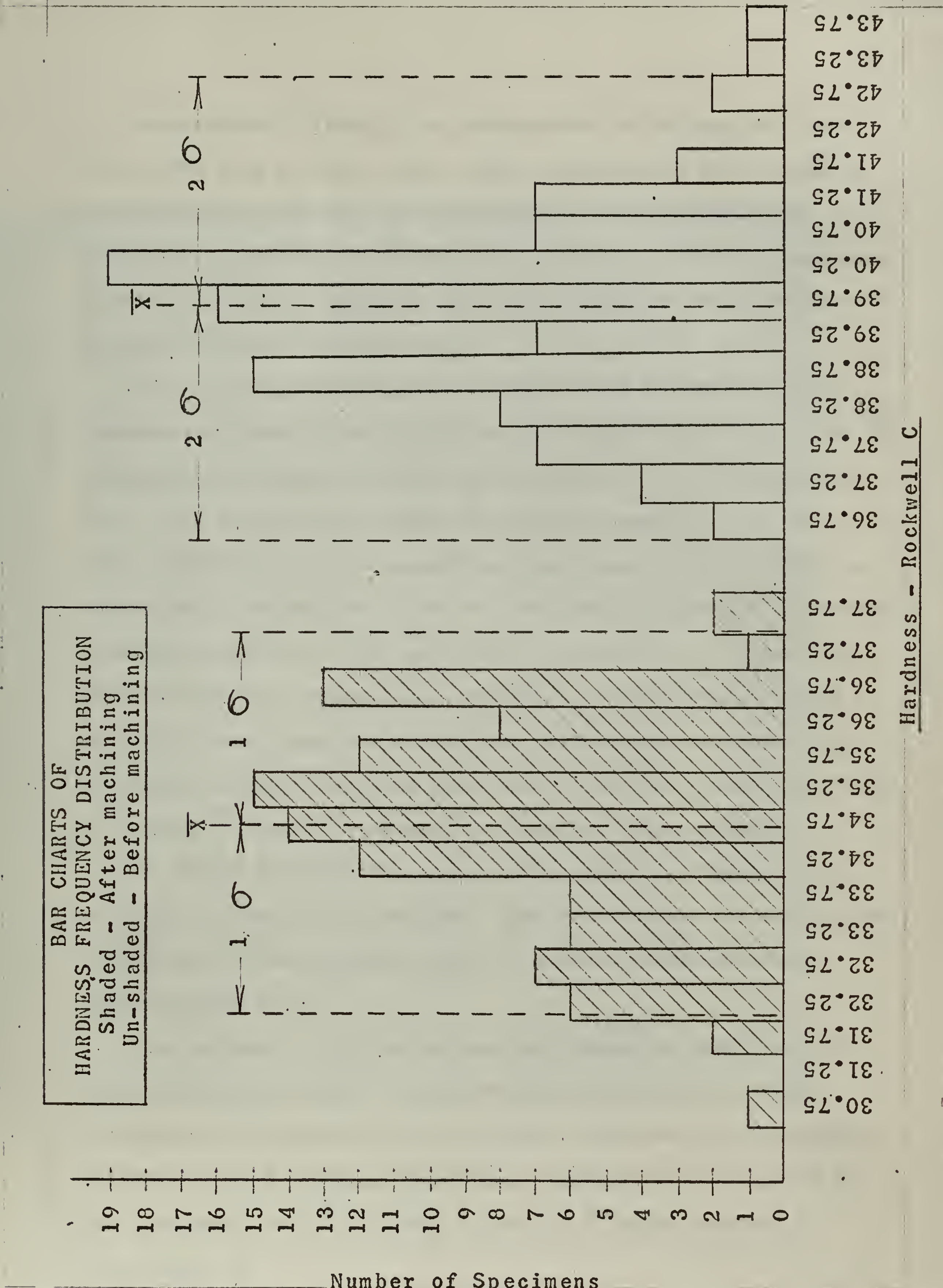


Figure 4 a) Upper chart b) Lower chart

Considerable difficulty was encountered in drilling the holes because the hole is quite small, and the material is hard enough to be difficult to machine. It was decided to use a new drill for each specimen in an attempt to obtain holes as nearly uniform as possible. In some cases it was necessary to use more than one drill because of breakage or dulled cutting edges.

The polishing operation was carried out in accordance with a procedure outlined in the Krouse Testing Machine Instruction Book [5]. Although a high degree of polish was desirable, it was emphasized that it was equally as important to have the same finish on each piece. With the aid of a magnifying glass under a strong light, an inspection of the quality of the surface finish was made. If circumferential scratches or tool marks were detected, the specimen was further polished. Longitudinal scratches, or marks mainly caused by the polishing operation itself were observed on some specimens, but only pronounced ones were sent back for further polishing as it is thought [8] they do not serve to initiate fatigue cracks.

It should be noted that the specimens shown in Figure 5 were not used as actual test specimens. They were rejects for one reason or another, and are intended only as a picture of the four basic configurations used.

From Appendix A, it can be seen that there are two values of specimen hardness listed; one taken after final heat treatment and before final machining, and one taken after the final machining and before actual testing. Based upon averages for one hundred and five specimens, this difference amounts to 5 points Rockwell C (see Figure 4).

Figure 5


A
B

As noted previously, the specimens had been left 0.030-0.040 in. in diameter oversize, and the final machining operations consisted of reducing the diameters by this amount, using two cuts at a moderately fast speed.

The test procedure was carried out as planned, after which time additional hardness tests were performed. Representative samples of all 4 types plus 2 rejected specimens which had not been tested for fatigue properties, were tested for hardness at sections A-A and B-B (see Figure 2) across two mutually perpendicular diameters. The Rockwell C hardness of the core was found to be 39-40, but at the edges dropped off markedly to approximately that of the surface, with an average value of 35.

As a final check, one of the specimens was prepared for a microscopic examination to see if there was any discernable difference in the micro-structure between the core and the outer circumference. The examination did not disclose any such difference, and it was thought that the final machining operation must have imposed a surface annealing effect upon the specimens.

4. TEST PROCEDURE

Test procedure A-2 as outlined by the American Society of Testing Materials' Committee E-9 on Fatigue [9] was the method followed in the determination of the fatigue data presented herein. This method consists of testing several groups of specimens at various stress levels. In order to estimate the variability of the data, it is recommended that at least 4 specimens constitute a group. At least 3 stress levels should be investigated to determine the probability-stress-cycle curves. In accordance with these suggestions, it was the original intention to use 5 specimens per group at 5 different stress levels for each configuration. This was not strictly adhered to in all cases because of specimens rejected due to faulty machining or other reasons given elsewhere.

The results from testing type A specimens (See Figure 5) were used as the standard for comparison with the other three types. General information is available in the literature concerning reduction in strength caused by holes [10] and surface finish [11], but it was considered necessary to evaluate these factors as accurately as possible for the material used, so as to be able to make a meaningful comparison with the final set of specimens in which the factors are combined.

The Brinell hardness corresponding to 40 R_c is approximately 370, and within this range the tensile strength of steels is very closely one half times the Brinell hardness, or 185 ksi. No tensile tests were conducted to verify this figure. It was de-

sired to have the material as hard as possible and yet still retain useful machineability in order to increase the notch sensitivity effect of the coarse machined surface in producing a relatively high equivalent stress concentration factor. This point is illustrated in Figure 6.

For steel, with Brinell hardness less than 400, the endurance strength has been found to be approximately one half the tensile strength. Accordingly, when testing type A specimens, it was assumed that the endurance strength was in the vicinity of 85-95 ksi.

The specimen diameters were measured using a thousandth-reading micrometer, with spherical anvils for the polished specimens and cone-shaped anvils for the coarse machined ones. They represent an average of two readings taken ninety degrees apart; the fourth decimal place has been estimated.

Specimens were chosen for stress level groups according to diameter. Because of excessive variation in diameters (the extremes were 4.8% smaller and 3.6% larger than the specified limits), it was not possible in all cases to have specimens in a group exactly the same (see Figure 7). An approximate bending moment for a group was determined according to the stress level under consideration using the formula:

$$M = \frac{\pi}{32} d^3 S \qquad \text{I}$$

where M is in inch-pounds if d is in inches and S is in pounds per square inch. Then, the nearest whole number bending moment was taken and the stress was computed for each specimen in the group. If, because of variation in diameter this resulted in an appreciable departure from the desired stress level, the necessary bending

moment was recomputed for the individual specimen. The resulting variation in stress was less than $\pm$ 1% (except for the type A group at 120 ksi with a stress variation of $\pm$1.19%), which apparently had no ill effect upon the results.

A total of 24 specimens of type A were tested to determine its probability-stress-cycle curves. In addition, the 6 specimens which had not failed after 5 million cycles were used again at the higher stress levels of 110 ksi and 100 ksi to help fill in the curve. One specimen had previously been tested at each of these levels in an attempt to get the "feel" of the curve so as to make advantageous use of the available specimens. The addition of three more specimens at these levels resulted in four points which, as stated previously, is the minimum recommended for each group. The decision to re-use the 6 specimens was based upon an investigation by Kommers [12] who noted that there appears to be very little effect upon the material structure at values of overstress less than 10% of the endurance strength (only 6% above in this case). Although he also noted that the effect of understressing was to increase the endurance strength, this effect is not very noticeable at 5 million cycles.

Type B specimens were the next ones tested. From Figure 6, based upon a hardness of about 370 BHN, there is a reduction in the fatigue strength of 73%, which is equivalent to a stress concentration factor of 1.37. This would put the endurance strength in the vicinity of 59 ksi. The curve of Figure 6 marked "Mirror Polished" is the standard for comparison of machined surfaces and represents

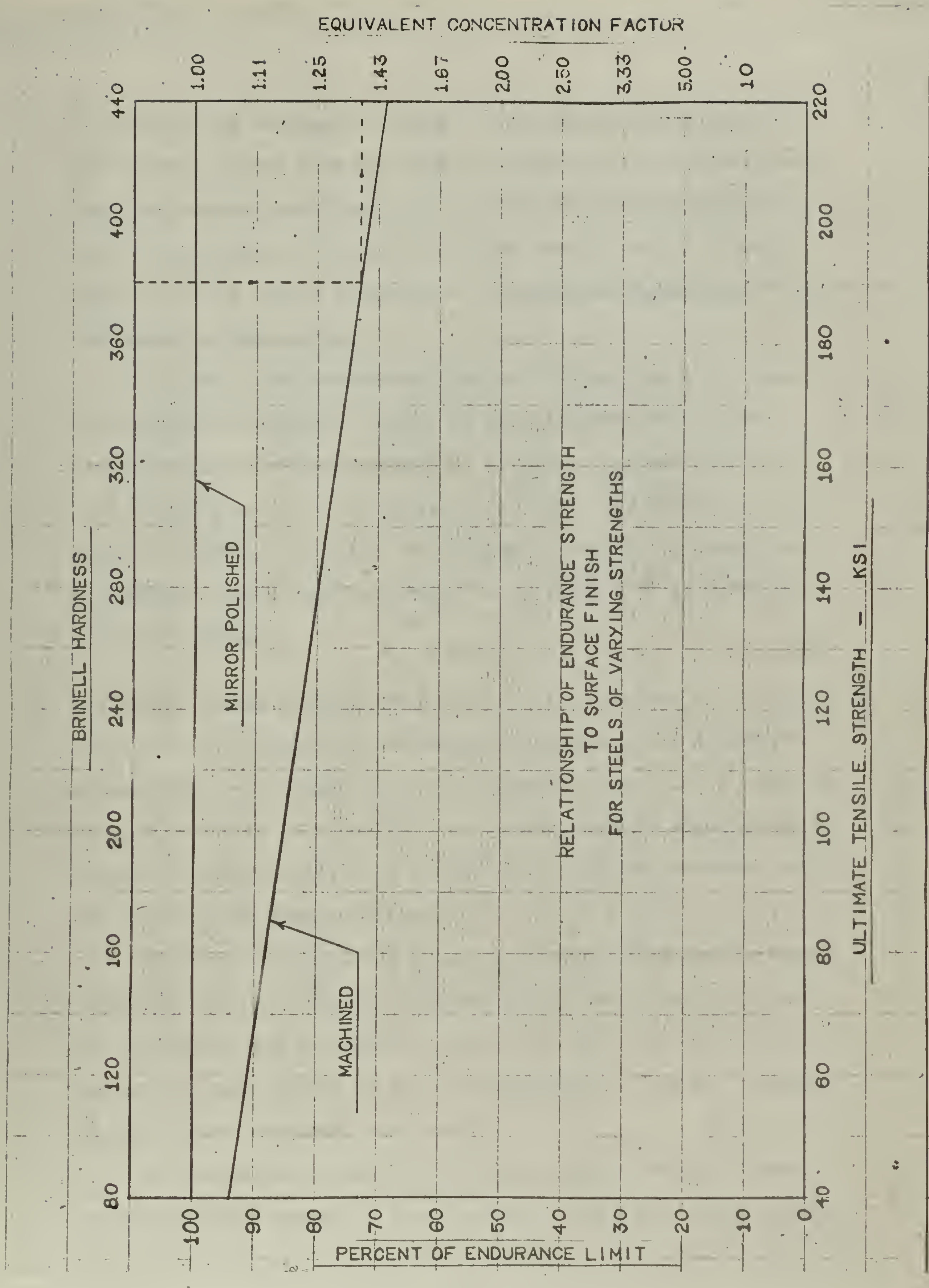

EQUIVALENT CONCENTRATION FACTOR
1.00
1.11
1.25
1.43
1.67
2.00
2.50
3.33
5.00
10
BRINELL HARDNESS
80
120
160
200
240
280
320
360
400
440
MIRROR POLISHED
MACHINED
RELATIONSHIP OF ENDURANCE STRENGTH
TO SURFACE FINISH
FOR STEELS OF VARYING STRENGTHS
ULTIMATE TENSILE STRENGTH — KSI
40
60
80
100
120
140
160
180
200
220
PERCENT OF ENDURANCE LIMIT
100
90
80
70
60
50
40
30
20
10
0

Figure 6

no reduction in endurance strength. Even though the specimens used in this report were not mirror polished, it is felt that any resulting stress concentration due to the difference between the mirror polish and the 10 micro-inch RMS surface finish is small enough to be of little consequence. Twenty-four specimens of this configuration were tested.

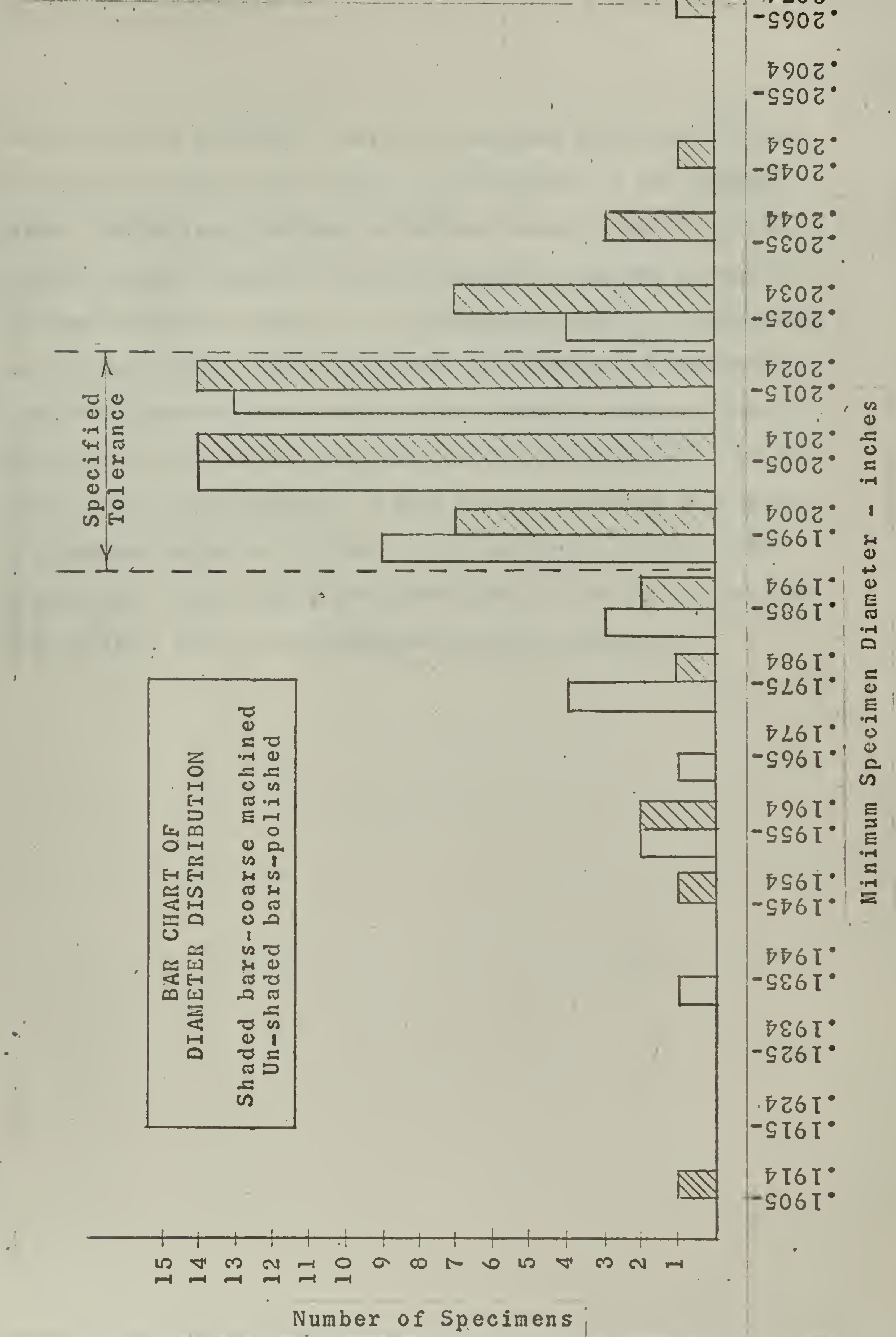
The third group tested was type C. The theoretical stress concentration factor for a ratio of hole diameter to minimum section diameter of the specimen of 0.125 was estimated to be 2.19 from Figure 8. (This Figure illustrates the procedure for a ratio of 0.130). The notch sensitivity factor, q, for quenched and tempered alloy steel according to Peterson [10] is 0.76, and using the formula:

$$K_f = 1 + q(K_t - 1) \qquad \text{II}$$

the actual stress concentration factor was determined to be 1.91. Based upon the results of the standard specimens, the endurance strength of these should be in the vicinity of 40 ksi. A total of only 19 specimens of this type were tested, as 6 of them had to be discarded because drills had broken off inside the specimens before the drilling had been completed.

The remaining and final group of specimens tested were type D. According to Guhse [4], the combined factor should be less than the product of the individual factors. At the very lowest the endurance strength should be in the neighborhood of 25 ksi. Twenty-eight of these specimens were tested.

The probability-stress-cycle curves shown in Figure 9 were fitted using the method of least squares. The lower limit of each



BAR CHART OF DIAMETER DISTRIBUTION
Shaded bars-coarse machined
Un-shaded bars-polished
Specified Tolerance
Number of Specimens
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Minimum Specimen Diameter - inches
.1905-.1914
.1915-.1924
.1925-.1934
.1935-.1944
.1945-.1954
.1955-.1964
.1965-.1974
.1975-.1984
.1985-.1994
.1995-.2004
.2005-.2014
.2015-.2024
.2025-.2034
.2035-.2044
.2045-.2054
.2055-.2064
.2065-

Figure 7

band is the 84% survival curve, with the upper limit the 16% survival curve and the heavy center line the median or 50% survival curve. The median is defined as the middlemost value of an odd-numbered group arranged in order of magnitude, and the average of the two middlemost values for an even-numbered group. These curves are all based upon a confidence level of 50% which is customarily used when presenting p-s-n data. As an example, refer to Figure 9, and for the type A at 100 ksi, the life for 84% survival is 92,000 cycles. Now, if a statement is made that at least 84% of a group of specimens tested at a stress of 100 ksi would survive 92,000 cylces, then, based upon a confidence level of 50%, it is expected that at least 50% of such statements would be incorrect.

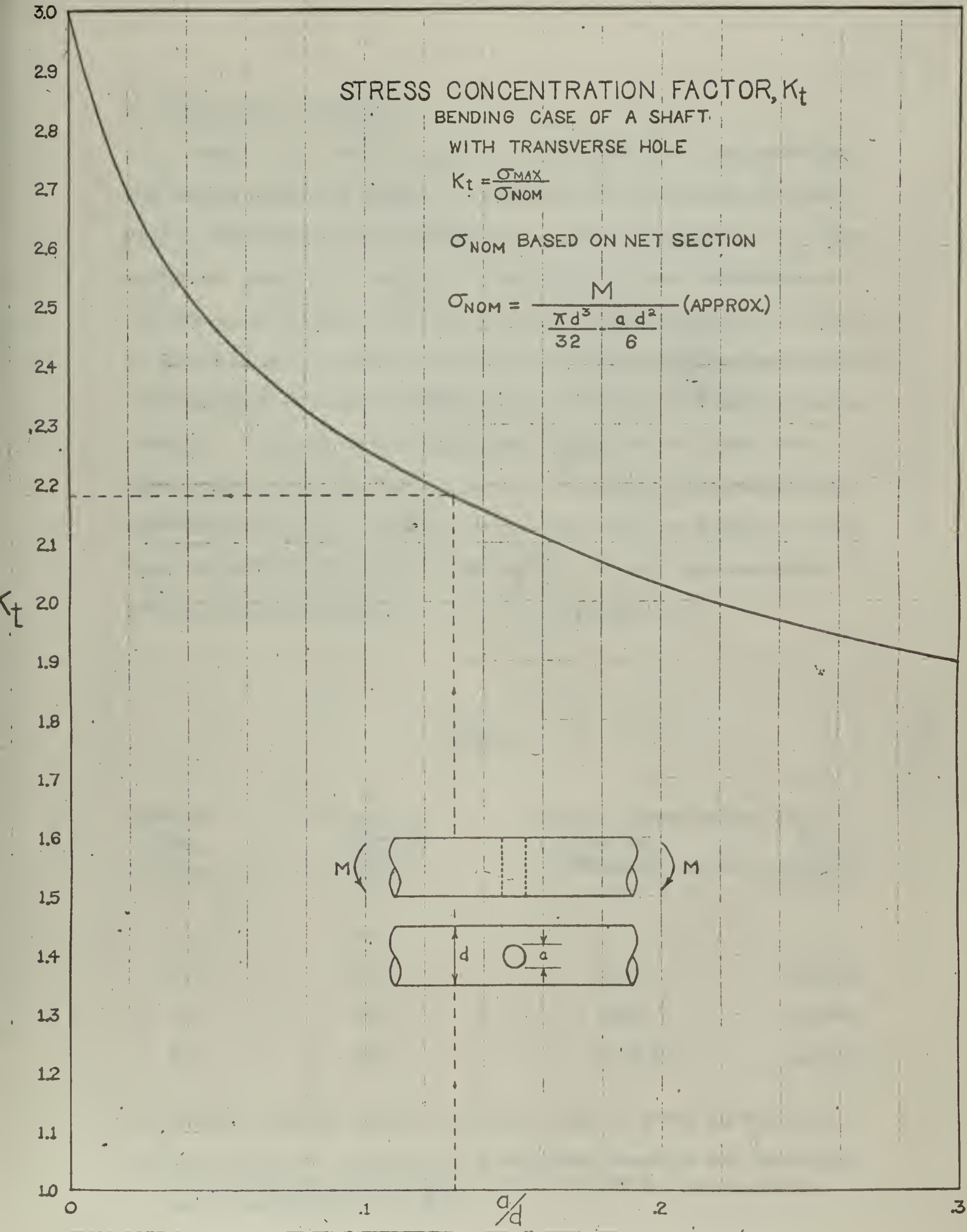

STRESS CONCENTRATION FACTOR, K_t
BENDING CASE OF A SHAFT
WITH TRANSVERSE HOLE
$K_t = \frac{\sigma_{MAX}}{\sigma_{NOM}}$
σ_{NOM} BASED ON NET SECTION
$\sigma_{NOM} = \frac{M}{\frac{\pi d^3}{32} - \frac{a d^2}{6}}$ (APPROX.)
M
M
d
a
K_t
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
.1
.2
.3
a/d

Figure 8

5. RESULTS AND DISCUSSION

Table A is a tabular summary of the results as obtained from the median curves of Figure 9, based upon the endurance strengths of the different types of specimens. Listed in column (1) are the endurance strengths. Column (2) represents stress concentration factors as determined from the literature (see discussion on pages 19 and 21), while column (3) lists the stress concentration factors as determined from this study, and are the ratios of the endurance strength of type A to the endurance strength of the other configurations. For the sake of clarity in further discussions, the numerically smaller stress concentration factor in column (3) has been designated as K_1, the larger as K_2, and the experimentally determined combined factor has been designated as K'.

TABLE A

Specimen Type (a)	Endurance Strength ksi	Stress Concentration Factors from the literature	Stress Concentration Factors this study
	(1)	(2)	(3)
A	82.7	- - -	- - -
B	54.5	1.37	$1.52 = K_1$
C	42.2	1.91	$1.96 = K_2$
D	34.8	2.78 (b)	$2.38 = K'$

(a) Capital letters refer to specimen type as shown in Figure 5.

(b) This value was obtained by multiplying together the individual stress concentration factors and represents a conservative and recommended value [11].

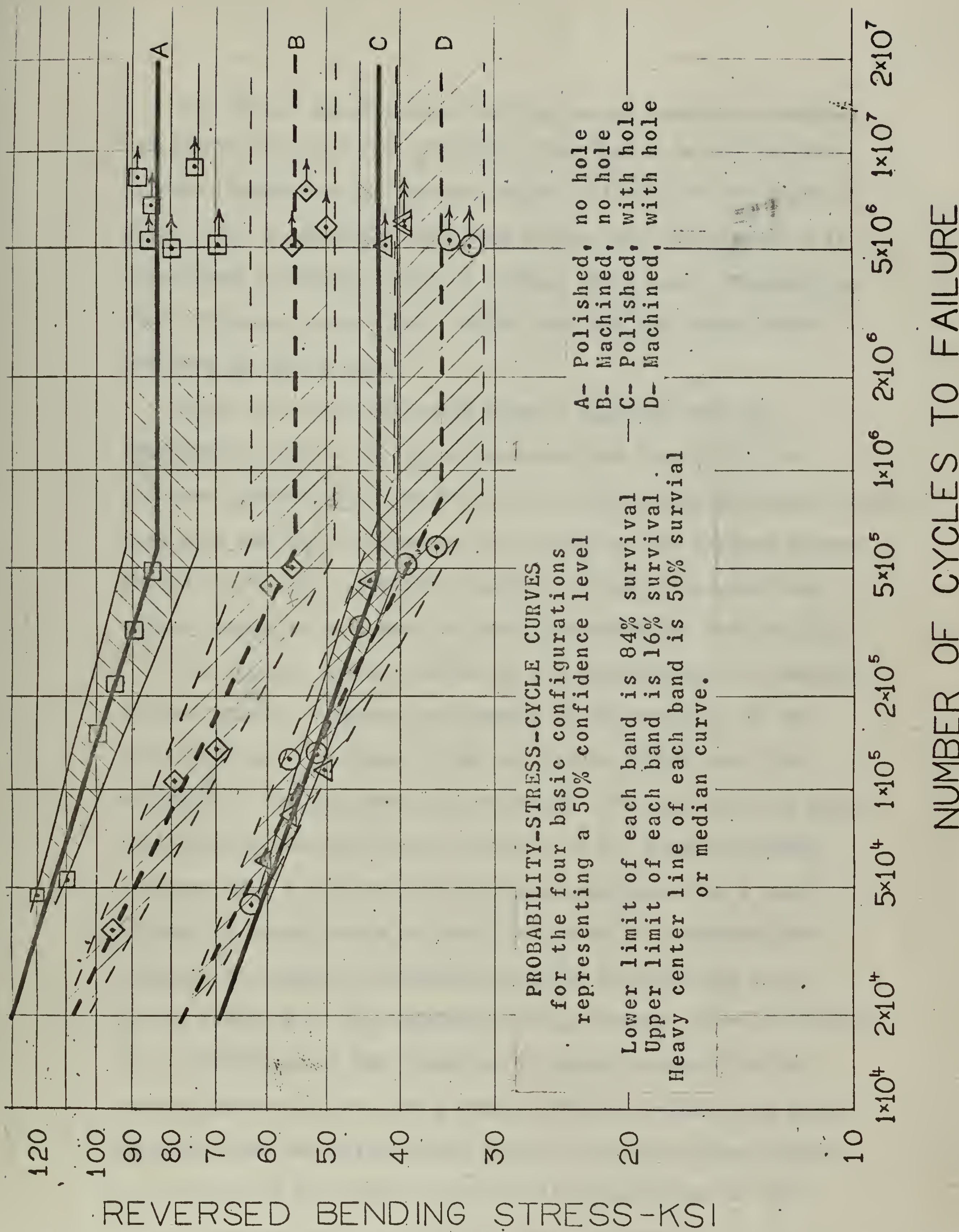

PROBABILITY-STRESS-CYCLE CURVES
for the four basic configurations
representing a 50% confidence level
Lower limit of each band is 84% survival
Upper limit of each band is 16% survival.
Heavy center line of each band is 50% survival
or median curve.
A- Polished, no hole
B- Machined, no hole
C- Polished, with hole
D- Machined, with hole
A
B
C
D
120
100
90
80
70
60
50
40
30
20
10
1×10⁴
2×10⁴
5×10⁴
1×10⁵
2×10⁵
5×10⁵
1×10⁶
2×10⁶
5×10⁶
1×10⁷
2×10⁷
NUMBER OF CYCLES TO FAILURE
REVERSED BENDING STRESS-KSI

Figure 9

The stress concentration for the coarse machined specimens was higher than had been predicted from Figure 6, and in fact, is even higher than the maximum value indicated on the chart of 1.45. This is probably explained by the fact that Figure 6 is based upon an average machined surface which has a roughness of about 250 micro-inches RMS, rather than the 500 micro-inches RMS used in this study.

There was close agreement between the predicted and experimental values of stress concentration factor for the polished specimen with the hole. Of all four configurations tested, this type had the least amount of scatter at the various stress levels, which could indicate that the effect of the hole was severe enough to mask some of the unaccounted-for variability.

The general overall effect of the superposition of the two discontinuities suggests that except in the vicinity of the endurance limit, surface finish had little effect upon the strength of the specimen with the hole. The fact that the graph indicates an increase in the strength of the coarse machined specimen with a hole over the polished specimen with a hole, within the stress range of 50-70 ksi, must be discounted and assigned to excessive variability and/or insufficient data. In the vicinity of the endurance limit, however, there definitely is an indication of the reduction in strength caused by the coarse surface finish. At a stress level of 39 ksi, two type C specimens had not failed after almost 6 million cycles, whereas all six type D specimens which had also been tested at the

same stress level failed with a median life of 522,000 cycles. Four more specimens tested at 35 ksi failed with a median life of 584,000 cycles.

Typical fracture patterns are illustrated in Figure 10. The polished specimens (A) give the appearance of having been pulled apart to a certain extent, and were quite violent in nature compared to the other types. The jagged structure of the fracture can be seen from Figure 10, and also the fact that it occurred in a plane about 30-40 degrees from the horizontal.

The machined specimens (B) illustrate the effect on the fracture of the circularly machined surface. They failed in a nearly horizontal plane and were not so rough as those of the standard specimens. It appears that once a fatigue crack had been initiated, it propagated itself along a circular tool mark causing the fracture to take place in that plane.

Fracture along the plane of the hole appeared to be the method followed by the standard specimens with the hole (C). Refer to Figure 10; failure seems to have started at one end of the hole (1), progressed around the circumference to point (2), and then since the cross-sectional area had been effectively reduced, the specimen pulled itself apart causing the relatively uneven portion of the fracture (3).

There is not much difference in the appearance of the fractures of the machined specimens with the hole (D) and without the hole (B), except that they were milder than all the other types and more uniform across the section.

If the individual stress concentration factors in column (3) of Table A are multiplied together, the result is $K_1K_2 = 2.98$. The experimental combined factor is $K' = 2.38$, a reduction of 20.1% from the product of the individual factors. This corresponds to 23.8% obtained from Guhse's [4] result for a similar combination. Using stress concentration factors as reported by Dolan [2], we find some confirmation of these values.

Table B is a compilation and comparison of the above results. A description of the contents of Table B is as follows:

Column (1) - the numerically smaller of the two individual stress concentration factors obtained by test.

Column (2) - the larger of the two individual stress concentration factors obtained by test.

Column (3) - the difference between the two individual stress concentration factors as a fraction of the larger.

Column (4) - the product of the individual factors obtained by test.

Column (5) - the experimentally determined combined stress concentration factors.

Column (6) - the percentage reduction of the experimental combined factor from the product of the individual factors.

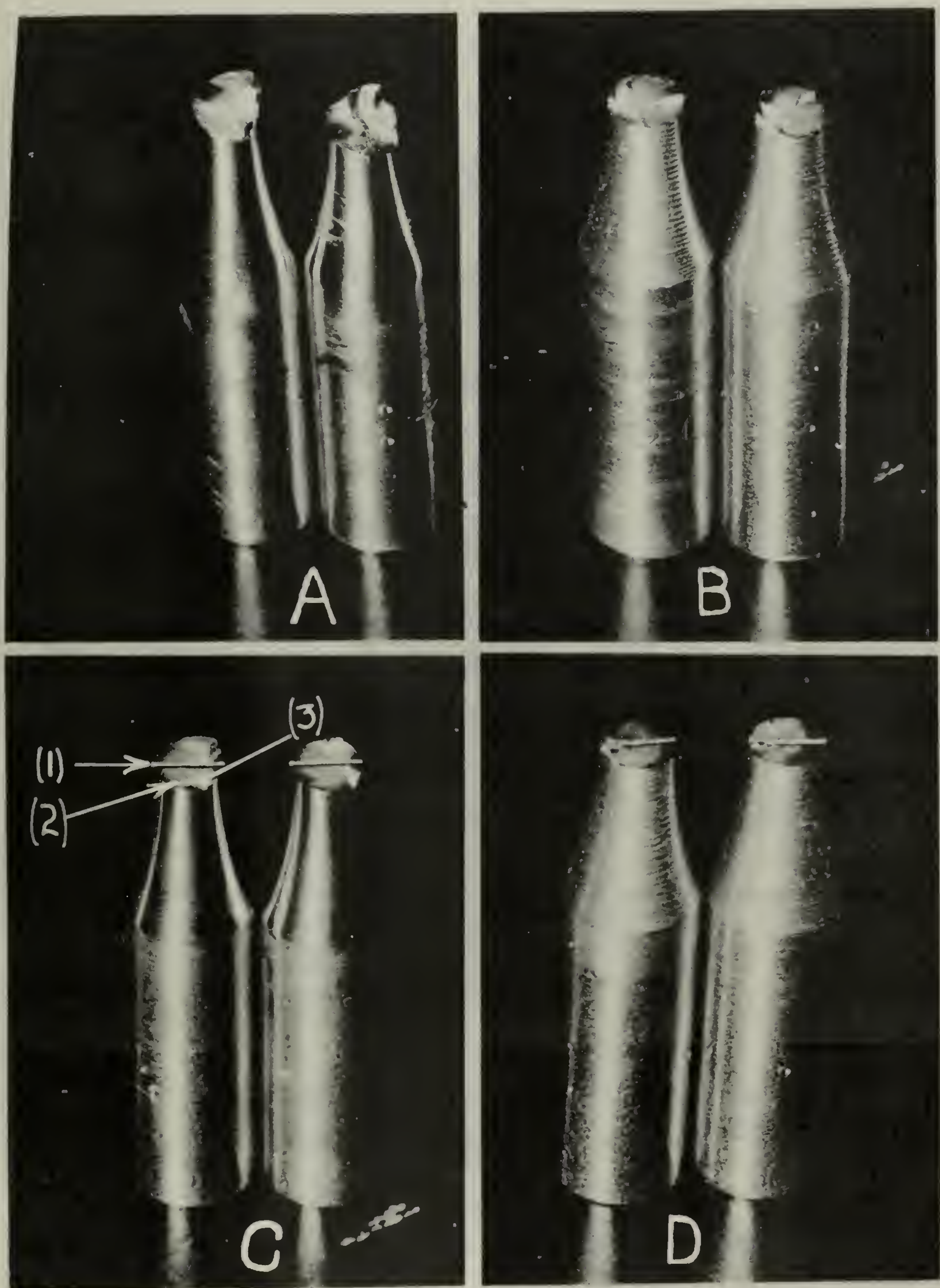


Figure 10

TABLE B

	K_1 (1)	K_2 (2)	$\frac{K_2-K_1}{K_2}$ (3)	K_1K_2 (4)	K' (5)	$\frac{K_1K_2-K'}{K_1K_2}$ (6)
Bridge (bending) 1*	1.52M**	1.96H	0.225	2.98	2.38	0.201
Guhse (torsion) 2	1.31M	1.83H	0.284	2.40	1.83	0.238
(torsion) 3	1.31M	1.44H	0.090	1.89	1.74	0.079
Dolan (bending) 4	2.90H	6.92C	0.582	20.07	10.00	0.502
(torsion) 5	1.72C	1.87H	0.080	3.22	2.80	0.130
(bending) 6	2.50F	6.92C	0.638	17.30	8.20	0.525

* The numbers in this column refer to plotted points shown on the graph of Figure 11.

** The capital letters in columns (1) and (2) refer to the particular discontinuity, as follows:
M- machined surface
H- hole
C- corrosion
F- fillet

An illustration of the results of Table B are shown in Figure 11, where column (3) vs. column (6) of Table B has been plotted. This curve was originally constructed without point 6, which represents the combination of fillet and corrosion (all other

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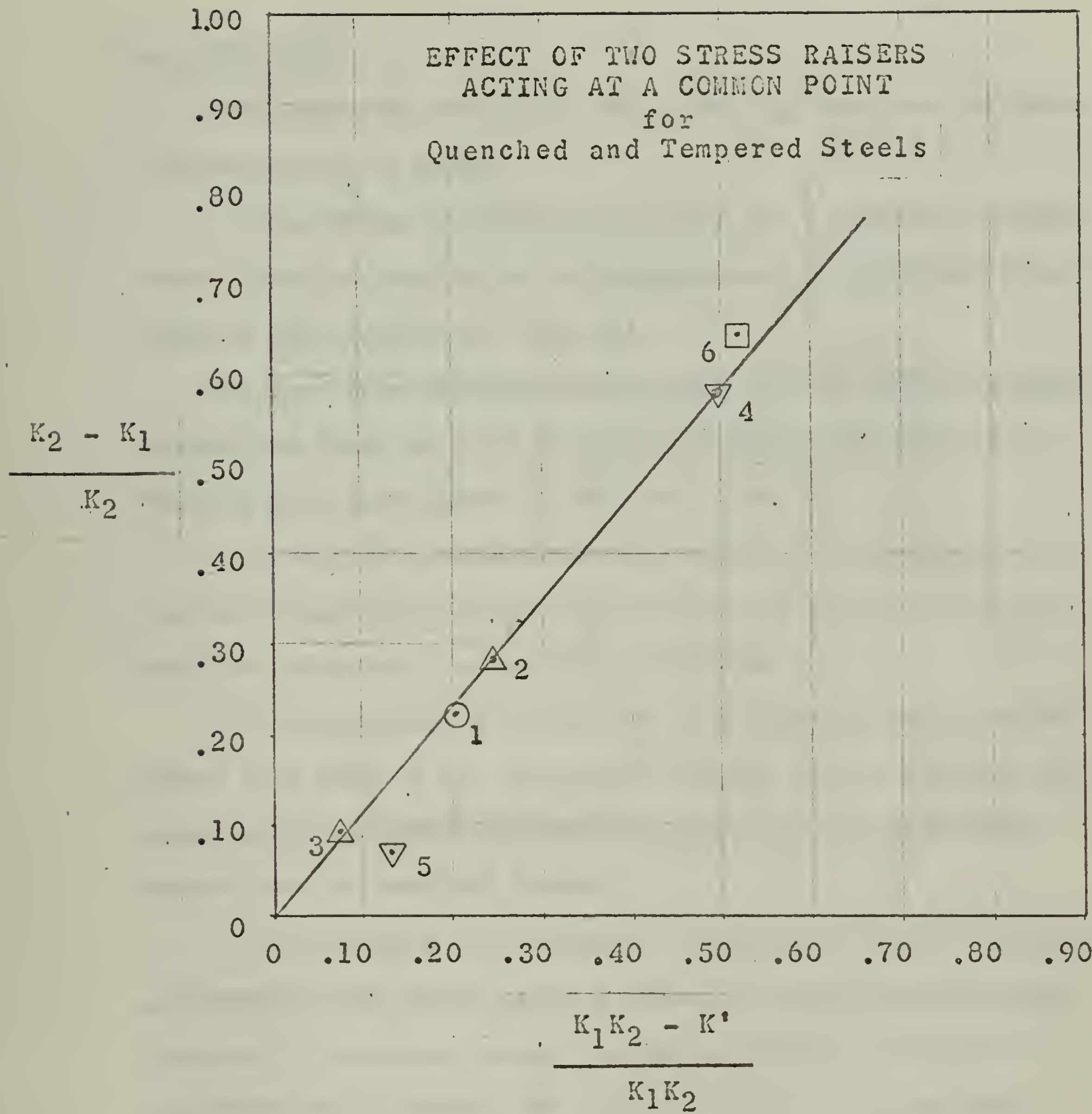
points have a hole as one source of stress concentration). It is seen that points 1, 2, 3 and 4 very nearly lie on a straight line through the origin, and suggests that for certain conditions a linear relationship may in fact exist. At least one known experimental point [2] shows a negative correlation with the curve of Figure 11 and thus is not shown.



1. This study, hole, machined, bending
2. Guhse, hole, machined, torsion
3. Guhse, hole, machined, torsion
4. Dolan, hole, corrosion, bending
5. Dolan, hole, corrosion, torsion
6. Dolan, fillet, corrosion, bending

Figure 11

6. CONCLUSIONS

Based upon the results of this investigation, the following conclusions may be drawn:

a) The stress concentration factor for a polished specimen with a hole is verified to be approximately as predicted from Figure 8 and Equation II, page 19.

b) The stress concentration factor for the coarse machined surface was found to be higher than had been anticipated from Figure 6 for reason given in Section 5 page 25.

c) The stress concentration factor for the combination of the coarse machined surface and the hole was found to be less than the product of the individual factors.

d) A comparison of these data with those of other investigators (see Table B and Figure 11) suggests that a straight line relationship may exist between the product of the individual factors and the combined factor.

It is recognized that conclusive proof of a general relationship between two stress raisers acting at a point has not been presented; there are so many variables involved. Figure 11 represents only quenched and tempered steels of approximately the same strength. According to Dolan's [2] results, Figure 11 does not hold for hot rolled steels. Also, from a study conducted by Mowbray [3], it appears that this relationship does not hold where the smallest individual stress concentration factor is close to unity. Mowbray used a specimen with a fillet which resulted in a stress concentration factor of only 1.03.

The smallest one represented in this paper is 1.31, reported by Guhse [4] for the case of a rough machined surface.

Further investigation should be carried out to determine the restraints which should accompany Figure 11, especially in the vicinity of points 4 and 6 of Figure 11 and higher. This is the area where the actual combined stress concentration factor K' is some 50% below K_1K_2; this difference could prove to be valuable to designers, if a degree of reliability can be achieved.

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APPENDIX A, TABULAR DATA

I-A. POLISHED SPECIMENS, NO HOLES

Specimen No.	Diameter inches	Hardness R_c (a)		Stress ksi	Cycles to failure
(1)	(2)	(3)	(4)	(5)	(6)
22	.2004	37.0	39.9	118.9	4.55×10^4
21	.1998	37.0	40.7	120.0	4.71×10^4
16	.2004	36.8	40.5	118.9	4.73×10^4
1	.2000	36.8	42.8	119.9	7.01×10^4
12	.1988	37.2	40.1	121.8	9.66×10^4
11	.2012	35.8	39.9	110.0	4.82×10^4
6	.1980	36.4	39.0	109.8	4.92×10^4
17	.2024	34.9	37.4	109.3	5.63×10^4
24	.2016	34.5	38.8	109.4	6.05×10^4
23	.1937	36.7	39.6	99.5	7.94×10^4
3	.2006	36.7	40.1	99.7	1.05×10^5
13	.1981	35.3	37.5	99.6	1.94×10^5
4	.2032	35.8	43.5	99.5	4.87×10^5
20	.2010	36.2	39.9	94.0	1.96×10^5
19	.2009	35.8	39.8	94.2	1.99×10^5
15	.2011	37.0	41.1	93.9	2.22×10^5
10	.2008	37.0	39.5	94.3	3.88×10^5
25	.1960	36.4	40.6	90.6	1.56×10^5
18	.1964	35.3	40.3	90.1	2.28×10^5
14	.1971	37.0	40.6	89.1	4.06×10^5
102	.1978	35.7	40.9	89.5	5.38×10^5
7	.2020	35.9	39.3	84.0	1.94×10^5
9	.2028	35.6	38.6	84.2	1.94×10^5
8	.2015	36.6	40.1	84.6	4.90×10^5
24	.2016	34.5	38.8	84.5	5.22×10^6*
17	.2024	34.9	37.4	88.4	8.21×10^6*
11	.2012	35.8	39.9	85.0	6.72×10^6*
4	.2032	35.8	43.5	80.1	5.00×10^6*
13	.1981	35.3	37.5	74.7	8.85×10^6*
6	.1980	36.4	39.0	69.5	5.06×10^6*

(a) Column (3) consists of hardness readings taken after final machining and before testing. Column (4) refers to hardness after heat treatment and before final machining.

* These specimens did not fail after being stressed 5 million cycles or more.

[illegible]

I-B. MACHINED SPECIMENS, NO HOLES

Specimen No.	Diameter inches	Hardness R_c		Stress ksi	Cycles to failure
(1)	(2)	(3)	(4)	(5)	(6)
32	.2031	30.7	38.8	94.8	2.76×10^4
48	.2030	32.9	40.1	95.0	3.20×10^4
27	.2038	33.5	39.9	95.0	3.68×10^4
45	.2032	33.0	40.4	94.7	5.49×10^4
29	.2009	35.8	41.4	79.1	7.17×10^4
34	.2009	34.3	39.1	79.1	9.98×10^4
49	.2007	34.8	39.2	79.4	1.15×10^5
50	.2004	35.8	39.8	79.7	1.33×10^5
38	.2010	34.4	37.7	70.2	9.80×10^4
40	.2012	34.6	38.4	70.0	1.28×10^5
39	.2013	34.7	37.6	69.9	1.42×10^5
41	.2014	34.9	38.8	69.8	2.38×10^5
42	.2018	34.4	39.6	59.5	2.56×10^5
43	.2016	35.2	39.7	59.7	3.91×10^5
35	.2020	34.2	38.6	59.3	4.86×10^5
47	.2019	35.9	40.0	59.4	1.17×10^6
37	.2040	34.3	36.9	55.2	2.58×10^5
30	.2024	36.4	41.9	55.3	3.96×10^5
33	.2025	33.7	38.8	55.2	5.05×10^5
46	.2026	35.2	40.2	55.1	8.72×10^5
44	.2028	34.3	38.8	54.9	5.07×10^6*
26	.1978	35.4	39.4	52.7	7.52×10^6*
31	.1906	34.5	38.2	50.0	5.80×10^6*

[illegible]

I-C. <u>POLISHED SPECIMENS, WITH HOLES</u>

Specimen No.	Diameter inches	Hardness R_c		Stress ksi	Cycles to failure
(1)	(2)	(3)	(4)	(5)	(6)
68	.2004	34.1	38.1	60.7	5.45x10^{4}
54	.1982	32.8	38.0	60.2	6.01x10^{4}
71	.2008	33.0	38.3	60.4	6.14x10^{4}
53	.2014	32.6	38.8	59.8	6.20x10^{4}
73	.2010	34.9	38.5	55.2	7.49x10^{4}
69	.2008	35.5	38.9	55.3	7.72x10^{4}
57	.2010	36.4	38.8	55.2	8.97x10^{4}
52	.2009	35.3	38.1	55.3	9.57x10^{4}
51	.2023	33.6	37.5	50.4	1.13x10^{5}
65	.2024	32.1	37.8	50.4	1.14x10^{5}
74	.2026	32.4	38.3	50.2	1.18x10^{5}
75	.2027	31.5	38.4	50.1	1.26x10^{5}
67	.2004	35.5	39.6	43.0	3.04x10^{5}
56	.2000	35.5	39.5	43.3	3.75x10^{5}
59	.1998	36.9	40.2	43.4	5.37x10^{5}
66	.1992	36.4	39.3	43.8	6.88x10^{5}
62	.1989	34.6	37.4	41.4	5.10x10^{6}*
63	.2020	33.1	37.7	39.5	5.89~10^{6}*
70	.2020	34.7	38.4	39.5	5.98x10^{6}*

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I-D. MACHINED SPECIMENS, WITH HOLES

Specimen No.	Diameter inches	Hardness R_c		Stress ksi	Cycles to failure
(1)	(2)	(3)	(4)	(5)	(6)
91	.2022	35.4	38.9	62.8	$4.09x10^{4}$
87	.2024	34.1	40.0	62.6	$4.16x10^{4}$
90	.2014	34.2	40.3	62.3	$4.66x10^{4}$
93	.2000	35.8	41.4	62.4	$5.93x10^{4}$
83	.2006	35.3	41.6	55.5	$6.87x10^{4}$
97	.1960	36.0	40.2	55.4	$9.80x10^{4}$
79	.2022	35.4	40.6	55.4	$1.51x10^{5}$
81	.1989	36.1	40.0	55.6	$2.20x10^{5}$
77	.2006	32.7	40.5	51.7	$1.08x10^{5}$
78	.2007	33.3	40.7	51.6	$1.14x10^{5}$
99	.2010	32.0	40.8	51.4	$1.44x10^{5}$
94	.2008	33.8	41.6	51.6	$1.49x10^{5}$
76	.1994	33.6	40.2	45.0	$2.79x10^{5}$
104	.1998	32.7	40.4	44.7	$3.29x10^{5}$
101	.1997	32.3	40.4	44.8	$3.30x10^{5}$
92	.1996	33.2	41.4	44.8	$5.14x10^{5}$
80	.2019	32.7	40.9	39.6	$2.95x10^{5}$
100	.2020	34.3	39.8	39.5	$3.22x10^{5}$
82	.2020	32.5	39.9	39.5	$4.53x10^{5}$
89	.2008	35.3	40.2	40.2	$5.89x10^{5}$
98	.2020	33.2	40.1	39.5	$1.20x10^{6}$
95	.2021	34.2	42.5	39.5	$1.42x10^{6}$
84	.2023	35.2	41.3	35.7	$3.08x10^{5}$
86	.2072	32.1	39.8	35.5	$5.56x10^{5}$
88	.2052	32.0	40.3	35.4	$6.11x10^{5}$
85	.2043	35.8	41.3	35.8	$6.25x10^{5}$
61	.1932	35.3	37.6	33.9	$5.17x10^{6}$*
105	.1996	34.7	40.4	32.0	$5.09x10^{6}$*

[illegible]

II - STRESS-CYCLE COORDINATES OF MEDIAN POINTS

Type A

Stress ksi	Endurance cycles
119.9	4.73×10^4
109.6	5.28×10^4
99.6	1.50×10^5
94.1	2.11×10^5
89.8	3.17×10^5
84.5	4.90×10^5

Type C

Stress ksi	Endurance cycles
60.3	6.08×10^4
55.2	8.35×10^4
50.3	1.16×10^5
43.4	4.56×10^5

Type B

Stress ksi	Endurance cycles
94.9	3.68×10^4
79.3	1.07×10^5
70.0	1.35×10^5
59.5	4.39×10^5
55.1	5.05×10^5

Type D

Stress ksi	Endurance cycles
62.5	4.41×10^4
55.5	1.25×10^5
51.6	1.29×10^5
44.8	3.30×10^5
39.7	5.22×10^5
35.3	5.84×10^5

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APPENDIX B, SAMPLE CALCULATIONS

1. DETERMINATION OF STANDARD DEVIATION FOR FIGURE 4(a)

X	f	fX	X^2	fX^2
36.75	2	73.50	1350.5625	2701.1250
37.25	4	149.00	1387.5625	5550.2500
37.75	7	264.25	1425.0625	9975.4375
38.25	8	306.00	1463.0625	11704.5000
38.75	15	581.25	1501.5625	22523.4375
39.25	7	274.75	1540.5625	10819.3750
39.75	16	636.00	1580.0625	25281.0000
40.25	19	764.75	1620.0625	30781.1875
40.75	7	285.25	1660.5625	11623.9375
41.25	7	288.75	1701.5625	11910.9375
41.75	3	125.25	1743.0625	5229.1875
42.25	0	0	1785.0625	0
42.75	2	85.50	1827.5625	3655.1250
43.25	1	43.25	1870.5625	1870.5625
43.75	1	43.75	1914.0625	1914.0625
44.25	0	0	1958.0625	0
44.75	0	0	2002.5625	0
45.25	1	45.25	2047.5625	2047.5625
	100	3966.50		157587.6875

X= Hardness at cell midpoint, R_c

f= Number of specimens in each cell

$\bar{X} = \frac{\sum fX}{n} = 39.7$ $\qquad \bar{X}^2 = 1575.8769$

$\overline{X^2} = \frac{\sum fX^2}{n} = 1573.3123$

$\sigma = \sqrt{\overline{X^2} - \bar{X}^2} = 1.60$

$2\sigma = 3.20$

[illegible]

[illegible]

$$[illegible]$$

2. METHOD OF LEAST SQUARES USED FOR FIGURE 9 *

X_i	Y_i	X_i^2	X_iY_i
4.51	17.19	20.340	77.527
4.85	16.57	23.523	80.365
7.92	15.95	62.726	126.324
8.94	15.58	79.924	139.285
10.09	15.18	101.808	153.166
11.35	14.82	128.823	166.207
47.66	95.29	417.144	744.874

$\overline{X} = 7.943$ $\overline{Y} = 15.882$ $\overline{X^2} = 69.524$ $\overline{XY} = 124.146$

The equation of a straight line is of the form:

$$Y = mX + c$$

where $m = \dfrac{\overline{XY} - \overline{X}\,\overline{Y}}{\overline{X^2} - (\overline{X})^2}$, and $c = \dfrac{\overline{X^2}\,\overline{Y} - \overline{X}\,\overline{XY}}{\overline{X^2} - (\overline{X})^2}$

Using the above figures, the equation for the sloping portion of the curve representing the polished specimens in Figure 9 is:

$$Y = -0.31X + 18.36$$

* The X and Y measurements were made upon a large scale chart.

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$$[illegible]$$

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$$[illegible]$$

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3. <u>DETERMINATION OF STRESS VALUES</u> (see discussion, p. 17)

As an illustration, the values for type A specimens at a stress level of 90 ksi will be used (see Appendix A, p. 36).

a) Using $S = 90{,}000$ psi, determine M, for $d = 0.1960$ inches:

$$M = \frac{\pi}{32}(0.1960)^3(90{,}000) = 66.5 \text{ in.-lb.}$$

b) Use $M = 67$ in.-lb., and compute the stress S, for each diameter:

$$S = \frac{(32)\,(67)}{(\pi)\,(0.1960)^3} = 90{,}608 \text{ psi}$$

Similarly, for the other three specimens:

$S = 90{,}061$ psi ($d = 0.1964$ in.)

$S = 89{,}105$ psi ($d = 0.1971$ in.)

$S = 88{,}200$ psi ($d = 0.1978$ in.)

c) For the specimen with the diameter of 0.1978 inches, recompute the stress using $M = 68$ in.-lb., from which:

$S = 89{,}477$ psi

d) The total per cent variation is:

$$\frac{90{,}608 - 89{,}105}{90{,}608}(100) = 1.66\%$$

e) The per cent variation about an average stress is:

$$\frac{1.66}{2} = 0.88\%$$

[illegible]

[illegible]

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4. DETERMINATION OF MEDIAN NUMBER OF CYCLES

As an example, we will consider the group of type A specimens which were tested at a stress level of 90 ksi.

Specimen Number	Cycles to Failure
25	1.56×10^5
18	2.28×10^5
14	4.06×10^5
102	5.38×10^5

$$\text{Median} = \frac{2.28\times10^5 \quad 4.06\times10^5}{2} = 3.17\times10^5 \text{ cycles}$$